Coalgebraic Methods in Computer Science and Mathematics

Picture yourself sitting behind a computer: it has an internal state, given by, e.g., the contents of its memory cells. This state is not directly observable, but the screen provides certain observable information by means of specific operations (e.g., listing the names of all files created so far). A computer is also a dynamic system: its internal state can be modified (for instance, by creating a new file).

This complex view of systems, consisting of both observations and internal dynamics, is captured by coalgebras. They were only introduced in the early 1980s as formal duals of algebras, to deal with infinite data structures, but they have been around in disguise for much longer. In fact, many coalgebraic structures have been in use for a long time in various situations in both mathematics and computer science, although usually not identified as such. Only recently have they been recognized to form the underlying structure of various kinds of dynamic systems, automata, transition systems, infinite data types, object-oriented systems, formal power series, and even various classes of differential equations.

Coalgebra thus offers a wide and rather unique unifying perspective on various parts of mathematics and computer science, comprising parts of formal logic, analysis, combinatorics, control theory, and discrete mathematics on the one hand, and algorithmics, complexity theory, object- and component-based software engineering, and automated verfication, on the other. As such, the study of coalgebra leads to a better understanding of seemingly unrelated disciplines, opening unexpected possibilities for crossfertilisation between techniques stemming from different worlds.

By now, the subject has gained worldwide recognition and interest among computer scientists, logicians, and mathematicians alike. CWI has been one of the first to put this exciting and promising field of fundamental research on its agenda, and is playing a leading role in its further development.

The highly interdisciplinary character of the study of coalgebra is illustrated by the various ways in which CWI cooperates both internally and externally on coalgebra and its applications. Among others, this involves the use of coalgebra for the supervisory control of discrete event systems, as well as applications of coalgebraic techniques for reasoning about programming logics and (secure) communication protocols.